## Fill the boxes

## Materials

Poster (optional): DIN A3, PVC 5 mm


7 Pieces: DM or 3D printer prisms
Dimensions (in unit):
$1 \times 1 \times 2 ; 1 \times 1 \times 3 ; 1 \times 1 \times 4,1 \times 2 \times 2 ; 1 \times 2 \times 3 ; 1 \times 2 \times 4 ; 1 \times 3 \times 3$
Unit $=1,85 \mathrm{~cm}$ (if coloured DM) or 2 cm if they are made in wood or 3D printer

Board: DIN A3, PVC 5 mm double board: a base board and a cover board in which the silhouettes of the 4 boxes are cut.

Dimensions (in unit): $3 \times 12 ; 4 \times 9 ; 3 \times 12 ; 6 \times 6$


## Brief description

Seven brick-shaped pieces, four lidless boxes where you can fit them all, and instructions (not shown).

## The instructions

The instructions can be printed on an A3-sized laminated PVC.
We recommend though-provoking questions rather than a detailed step-by-step procedure:

- Could you fit all these pieces inside the box?
- Could you pack the pieces in a more compact disposition?
- What are the volumes of the pieces? And what about the box?
- Could you design other boxes that have the same volume?
- Which box uses less material?


## The pieces

All pieces are brick-shaped and are specified using a common "unit length" (we use 4 cm in our large wooden prototype,
but for a mobile format, $u=2 \mathrm{~cm}$ works better). Their proportions are:
A. $1: 1: 2$
B. $1: 1: 3$
C. $1: 1: 4$
D. 1:2:2
E. 1:2:3
F. 1:2:4
G. $1: 3: 3$

The lengths are not specified in the material and they are not written in the instructions, since we want to encourage the learner to
"discover" the proportions by comparing the pieces.
Note that the total volume of the pieces is 36 cubic units, which has many small factors, and that will play a fundamental role in the use of this material.

## The boxes

The windows of the upper layer of the board should be sufficient to suggest how to carry out the activity, which must in any case be understood as a work-in-progress, in which one must expect that the first proposals are different from the final objective that we propose and to which can be achieved gradually. The normal situation is that a previous participant has left one of the 4 structures built and this can represent a further and decisive suggestion for the next user.
The design of the windows is based on the same "unit length" used for the pieces. More specifically, the interior of the boxes must have the following proportions (the exterior size will depend on the thickness of the material used to make the boxes):
a. $1: 3: 12$
b. $1: 4: 9$
c. $1: 6: 6$
d. $2: 3: 6$

The first number (in boldface) is the height of each box. For practical reasons, it is very convenient to make the heights a little shorter (so the pieces protrude a bit from the boxes and, therefore, are easier to grab) and the other to dimensions a little larger (so the pieces fit inside the box with some slack and, therefore, are easier to pack).

## For 3D Printers

In the 3D printer version, the pieces will be designed with some carved details in basrelief, so they resemble suitcases or groceries. It is fundamental, though, that you can stack them easily and that their proportions are still deductible by inspection.

The boxes could also be designed to be 3D printed, although this will limit too much their size and, therefore, the size of the pieces: Some 3D printer beds are $15 \times 15 \mathrm{~cm}$ and the longest box is 12 units wide, so the "unit length" will become as short as 1 cm in those 3D printers.

Alternatively, the boxes can be replaced with 2D printed (same format as the instructions) rectangular "bases" resembling the trunk of a car, a cardboard box, or a fridge (depending on the final piece design) with the same proportions as the wooden boxes bases:
a. $3: 12$
b. $4: 9$
c. $6: 6$
d. $3: 6$


## Competences

- The notion of "unit length" and comparison-based deduction of the proportions of the pieces without using measuring tools.
- Computing the surface area of a 3D figure.
- Computing the volume of brick-shaped pieces and boxes.
- Notion of volume conservation and volume decomposition.
- Mental arithmetic: products and additions.
- The notion of divisor and the prime number decomposition.
- Combinatorial problems regarding composite number decomposition into (exactly) 3 factors.
- Minimization problem of figures of constant volume with minimal surface area.

