The Bill Paradox

Materials

Two DINA3 boards with a white rectangle on them. The pieces of the bills are in PVC or in wood to place them in the rectangle. There will be two paradoxes.

For the first one, for one side should appear the 100€ bill, but for the other, the 50€ bill.

For the second bill, on one side there's a 20€ bill and on the other, the 50€ bill.

Brief description

With the pieces of the exhibit you have to assemble the 100€ bill (5 pieces), but if you flip all the pieces back, then you can assemble the 50€ bill…but there's one piece left over!

For the second paradox, you will assemble the 20€ bill (3 pieces) and then you can flip the pieces and assemble the 50€ bill…and there's one piece gone!

There's a paradox, and the idea is to try to explain what's happening there.

Assembly

Design of all the pieces

The pieces are shown in the pictures with the dimensions.

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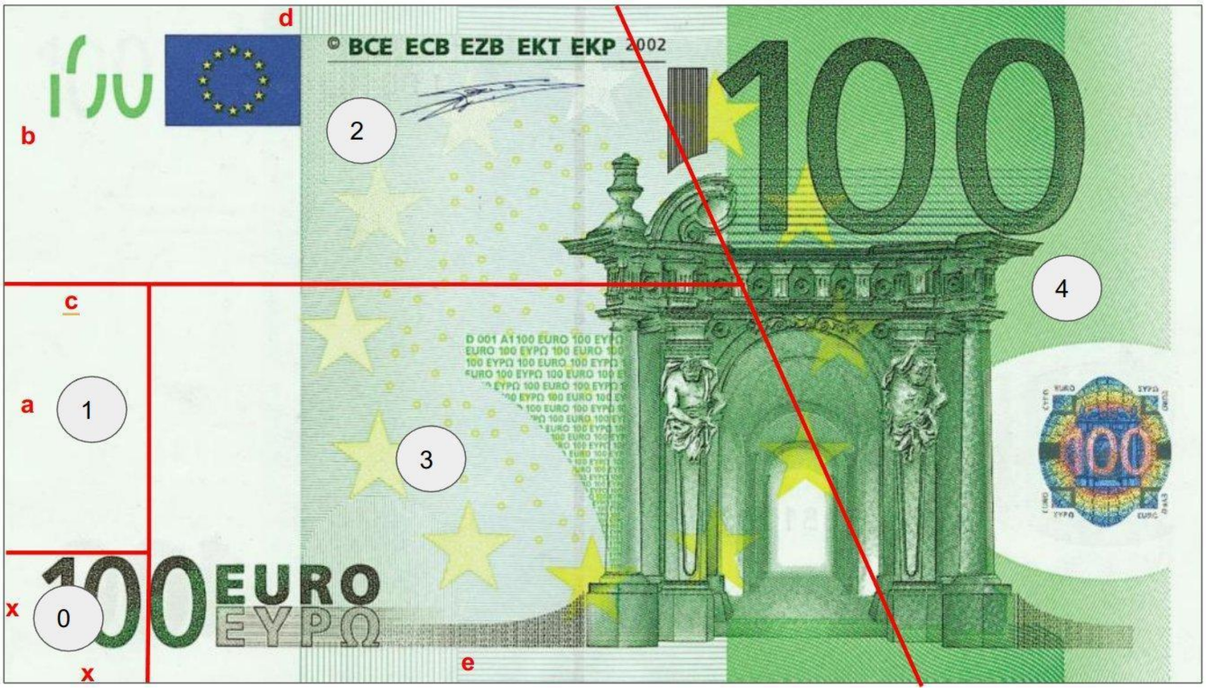
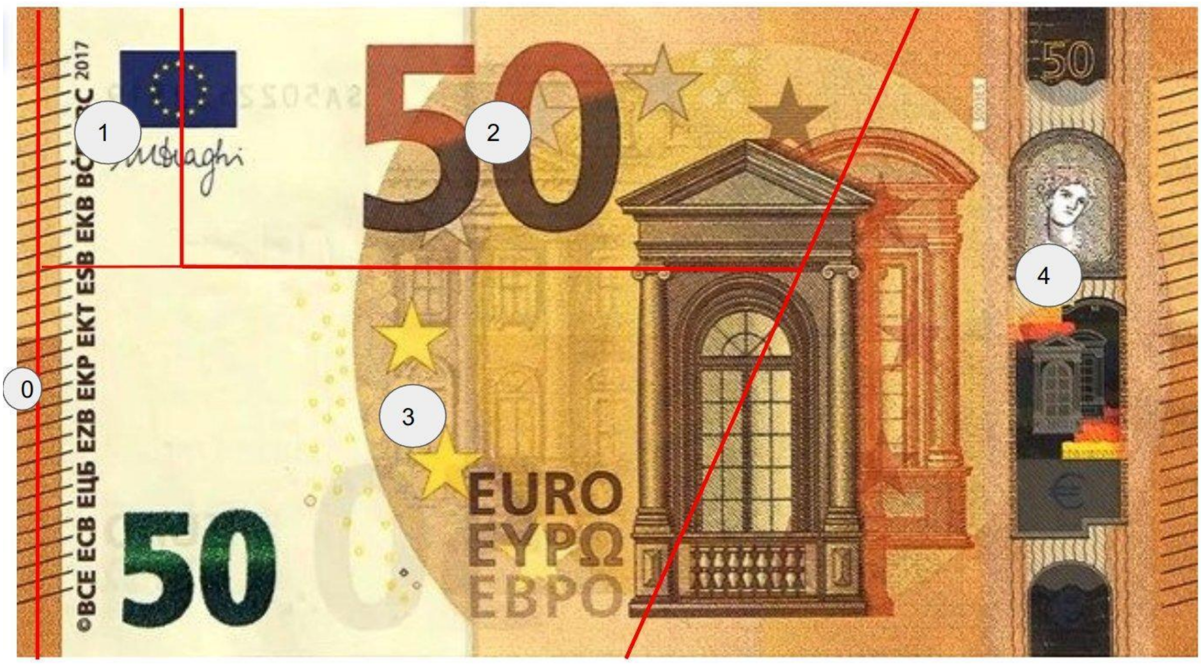
FIRST PARADOX:

Front of the bill:



Back of the bill:



(Neither the pictures of the bills nor the measures are in the proper scale, they are only for an explanation purpose)

Two condition must be satisfied regarding the measures of the pieces:

a = b

c + d = e

(the rest of the measures can be as desired)

For example, for a Bill Dimensions:

15cm x 8,5cm

Take:

a = 1,1 cm
b = c = 3,7 cm
d = 7 cm
e = 8,1 cm

The piece which disappears is the number “0”, so in the image of the 50€ bill you will have to cut the left and right border a little bit (exactly the area that is left over), so the difference is not noticeable.

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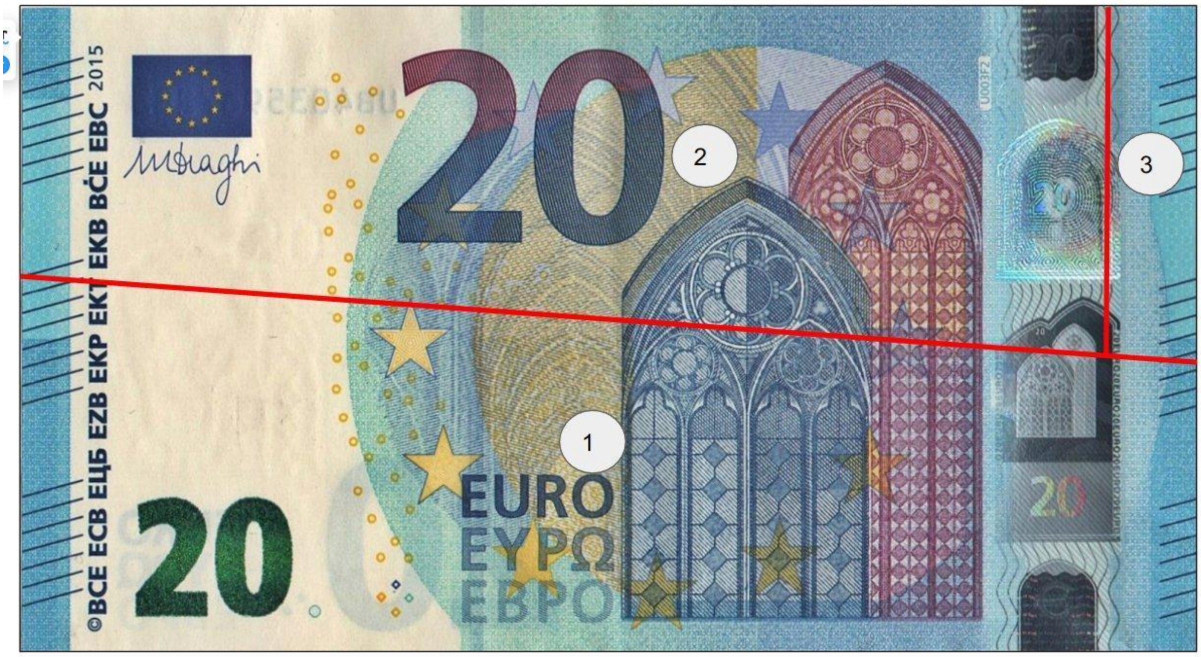
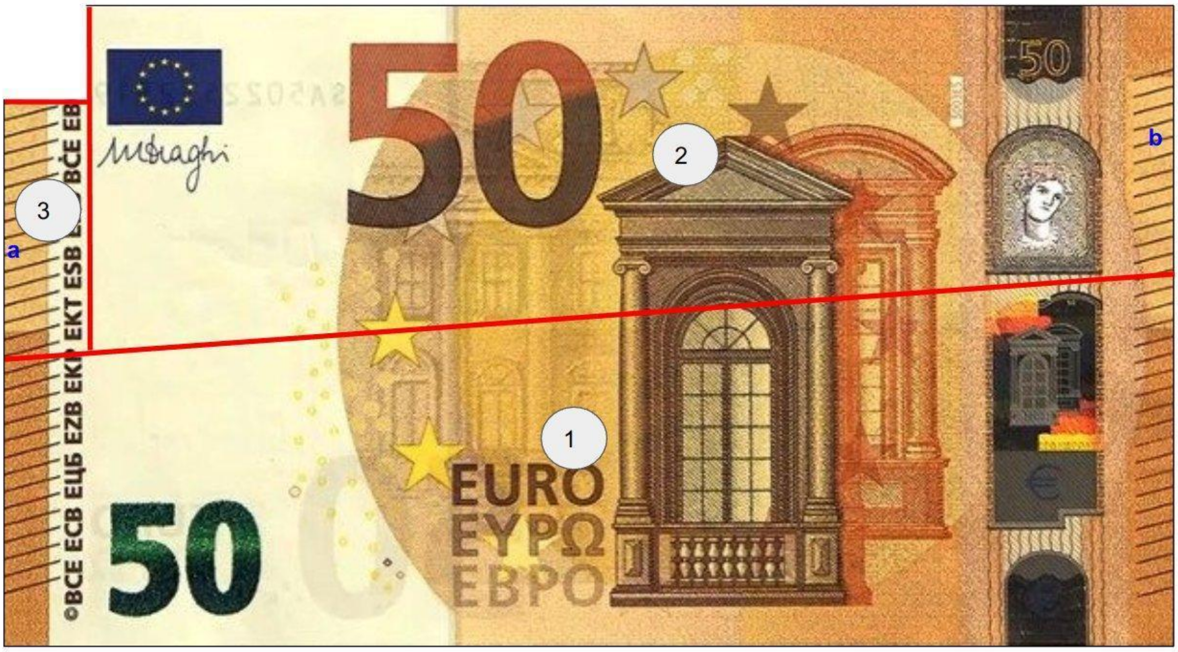
SECOND PARADOX:

Front of the bill:



Back of the bill:



There is a small square on the corner that “disappeared”. It is the same area that it increased in the bill of 50€. So, the 50€ bill it is slightly bigger but the difference it’s not noticeable to the eye.

Only one condition must be satisfied regarding the measures of the pieces:

a = b

(the rest of the measures can be as desired)

For example, for a Bill Dimensions:
15cm x 8,5cm

Take
a = b = 4,5 cm
c = 1 cm

Assembly

TIP: It’s extremely important the accuracy and being very precise in taking the measurements and making the cuts. The two paradoxes are assembled in the same way.

Made in PVC, print the 100€ for one side with the lines to cut. Print the 50€ in another paper with the lines to cut. Cut all the pieces and glue them back together. Two pieces with the same shape must be glued together. There is one piece left over: the squared one.

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The Board (DINA3)

The boards are quite simple, since it will have only a question and a white rectangle drawn with the same dimensions as the bills.

Text (board 1): “Assemble the 100€ bill with these pieces”. “Flip them all and assemble the 50€ bill”. “What happened? Why?”

Text (board 2): “Assemble the 20€ bill with these pieces”. “Flip them all and assemble the 50€ bill”. “What happened? Why?”

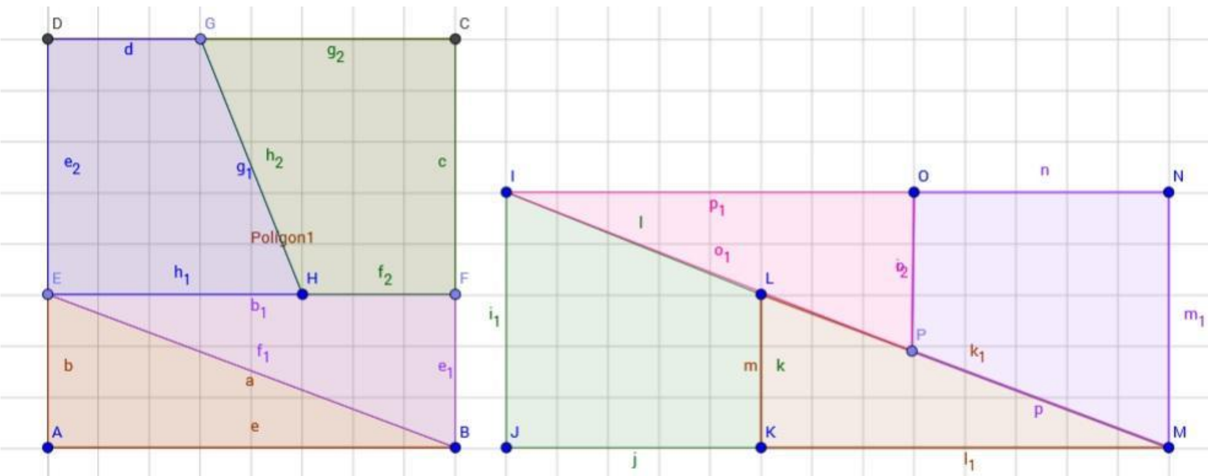
Other Options

There are other versions of the paradox using another topics like chocolate

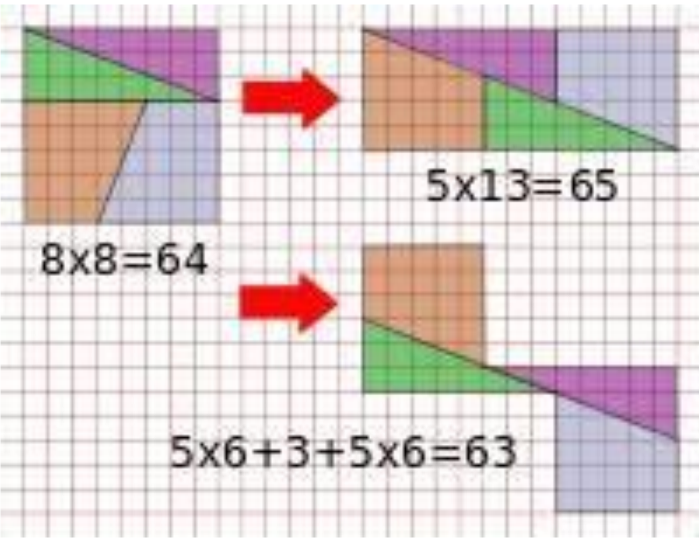
or a card trick:

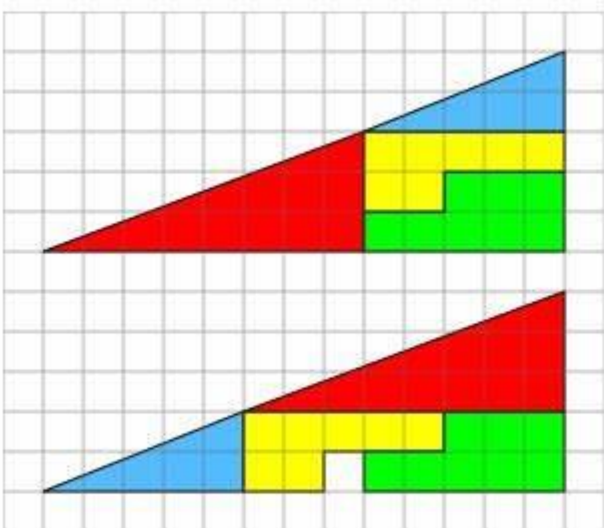
https://www.youtube.com/watch?v=BBrtzaZQeso

Another option, that can be developed in the classroom, is transforming a 8x8 square into a 13x5 rectangle, so that 64= 65. It can be easily dawned on a squared paper to be cut and reassembled. It involves Fibonacci numbers.



Another similar paradox is the Paul Curry's one:





Again, it plays with the Fibonacci numbers (1,2,3,5,8,13)

Explanation

The teaser introduces the idea of a paradox in mathematics. How is it possible to make the same shape with different areas?

For the first paradox, of course, the two bills don't have exactly the same area. The 100€ bill is a bit larger than the 50€, but the area of the square that is taken out, is shared for all the 50€ bill to make it similar to the 100€, so the eye can not notice the difference.

The same happens with the 20€ - 50€ paradox, but here the 50€ it's a bit bigger. The area of the disappeared square is taken for the upper and downsides of the 20€ bill.

The main idea is to provoke a surprise that will become a conversation among the visitors regarding some geometrical concepts: areas, shapes, properties, etc…

Competencies

Geometrical concepts can be involved in the conversation to give an explanation to what seems impossible, like the area of some quadrilaterals and mathematical paradoxes.

The fact of solving the teasers also needs some basic concepts as rotation, translation and symmetry.

Observations

As it said above, it's extremely important the accuracy and being very precise in taking the measurements and making the cuts.

For 3d Printers (If applicable)

The construction for 3D printers is almost the same as the above version. Since you cannot print ink directly, then you can (ink) print the pictures of the bills in a glue paper and add it afterwards. Although it is not necessary, since you will have a geometrical teaser when you can make a square disappear!

You can also add some textures to the pieces for SLD’s people.

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