## Hidden Coins

## Materials

Two types of 3D-printed coins, the 3D-printed box, bolts and nuts, screwdriver.

## Brief description

In this module, the players will have to find out the value of the money there is in the box. However, each time they shake the box, they'll only be able to see random samples of the coins within. With only that information, they'll have to answer the following question: There are 20 coins in this box. What is the value of the money in it?

## Assembly

## Design of all the pieces

Print files in 3D.

## Assembly

Put 20 coins in the box. Put the lid over the box. Screw together both parts of the box.


## The Board (DINA3)



## Hidden coins

Each of the 20 coins in the box is worth 1 or 2.
Can you find out, or estimate, the total amount


## Other Options

If you don't have a 3D printer, both the box and the coins can be made in other ways.

1. PVC

The box should be made out of two $30 \times 30 \mathrm{~cm}, 5 \mathrm{~mm}$ thick PVC squares attached to each other by Velcro. One of them should have a rectangular window as indicated in the image. For the coins, you can use any coin-like discs (for example poker chips or disks for the game 4 in a row)



If the distance between the top and the bottom of the box is x mm high, it will be necessary for the thickness of the "coins" to be greater than $x / 2+1 \mathrm{~mm}$ and less than $x-1 \mathrm{~mm}$ so that the "coins" slide inside the box without overlapping. In the case of poker chips, two should be glued together in order to be thick enough.

For the same amount of coins (5) to always be visible through the window, the lid must have a rectangular window no longer than 5 times the diameter of a coin and half a coin high, as shown in the previous image.


## 2. Bags

You need a bag, discs of 2 different colours for "coins" and a spoon. This is the easiest and cheapest solution for supervised players, but supervised players only, since the guide will have to ensure players don't cheat by looking within the bag.

It also allows for the guide to push variants of the game core system.

For example:

- 3 colors instead of two.
- more discs


3. German tank problem

Can you estimate the number of pieces in the bag by taking just one sample? Yes, if the pieces are numbered, like, for example, bingo balls. This is known as the German tank problem and was used during World War II to estimate how many tanks the German army had based on the serial number of a few captured tanks.

To get this estimation, the balls will have to be numbered from 1 to N , and a random sample ( $k$ ) of balls should be extracted. If $m$ is the largest number in the sample drawn, a good estimator of $N$ is $m \cdot(k+1) / k-1$ with a standard deviation of $N / k$ if $k$ is much less than $N$. This is a minimum estimator variance without bias. At the previous link to Wikipedia you can read how to calculate the $95 \%$ confidence interval if the $k$ samples are extracted one by one.

## Explanation

In every option, the visitor will have to use probability, rational numbers and equivalent fractions, since the coins in each sample may or may not be the same as the ones from the previous one.

This allows for a series of interesting questions to ask:

- How many shakes do you need to answer "properly"?
- How many coins do you need to have seen to be "sure" about the answer? What does "being sure" mean?

The most interesting part about it is that no one can possibly know the answer, not even by taking many samples. But statistics can teach a lot about it.

## Competencies

Probability (Laplace rule), statistics, proportions, fractions, approximations, calculus, problem-solving and adaptive strategy-making.

## Observations

The question on the board could be "How many of each type are inside?" if you use coloured discs.

